

Theories of Growth and Development Fall 2001, Midterm I

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YOU HAVE 3 HOURS FOR THIS EXAM. THUS TIME IS AN EXTREMELY SCARCE GOOD. USE IT **OPTIMALLY**

1) (5 points) Discuss analytically as an economist what you understand from the phrase above, that “time is scarce and you have to allocate it **optimally**”.

2) (30 points)

Consider the following private ownership economy: There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone.

The total wheat harvested is given by the (neoclassical) production function $g k q = .$ Currently 0.1 units of wheat is saved and invested as seeds for next year’s production cycle.

The population growth rate is zero in this economy. Bear in mind that in this model, seeds invested in the previous year become capital stock of the current year. Otherwise capital cannot be stored and accumulated. Thus depreciation of capital, d is 1.0. The technology parameter, g , is known to be 0.5.

a) Solve for the neoclassical long run equilibrium growth (the steady state equilibrium) values of the capital labor ratio, output per labor in this economy.

Solve for the gross profit rate $(r + d)$ and the wage rate under the neoclassical steady state equilibrium?

(i) Verify that the unit price equation holds for this economy: $1 = wl + (r + d)ky$

(ii) Using the classical saving function obtain the Marxian growth rate for this economy.

(iii) Using the demand-supply equilibrium ($1 = cl + gky$) find the per capita consumption level in the Marxian model. Show the Marxian equilibrium in a 4- quadrant graph, using the wage rate, consumption, growth rate and the gross profit rate values you have depicted above

(iv) Now find the per capita consumption level in the neoclassical model under the steady state. Show it with the aid of a graph on the $y - k$ space depicting the neoclassical steady state equilibrium.

(v) Contrast the neoclassical and the Marxian values of per capita consumption. How do you account for the differences?

b) Now consider the *Golden Rule of Accumulation* under neoclassical model. Find the new saving rate that would maximize per capita consumption per capita consumption under the steady state. Find neoclassical wage rate and the *net* profit rate (r) under the golden rule.

(c) Find the capital labor ratio and the wage rate and the net profit rate under the *Golden Age of Capital* that is the value of k that would maximize the net profit rate in this economy.

(d) Show the neoclassical steady state equilibrium values of the golden rule and the golden age. Is the capital labor ratio higher or lower in the Golden Age as compared to the Golden Rule? How do you account for this difference?

3) *Investing in Capital* (15 points)

Consider the following three countries, A, B, and C. Each country is endowed with the same amount of capital and labor, and the respective saving rates out of national product are the same. Suppose that the countries are also identical with respect to their technologies in producing the final good and they are initially at their initial steady states. Finally assume that the rate of population growth is the same across countries, and that there is no (exogenous) technical change occurring.

Suppose now that in each country a dictator assumes power and announces the following policies: *Country A* announces that it will increase its saving rate by a factor of two for the next ten years to “provide better living conditions for the next generations and to surpass the other two countries”. It plans to use its extra savings on investing physical capital. After the end of the ten-year policy the saving rate is planned to be set to its initial rate. *Country B* announces that it will also increase its saving rate by a factor of two for the next ten years to “provide better living conditions for the next generations and to surpass the other two countries”; and plans to invest on education and human capital formation –*i.e.* producing “educated labor force and knowledge”. After the end of the ten-year policy the saving rate is planned to be set to its initial rate, just like in *A*. *Country C* does not follow any change in its existing economic policies.

Discuss the nature of the transitional growth after the dictators assume power in all three countries. What will be the per capita rates and levels of growth in the respective countries at the new steady states? Discuss the position of the new steady states in comparison to the initial conditions. Use graphs or algebraic formulas, if necessary.

4) (10 points)

Consider the kaldorian model where workers do not save, and investment is given from outside with $I = I^*$. Show the share of wages in output and verify that it is inversely

related with the Investment share of aggregate demand.

5) (15 points)

a) Suppose that an economy obeying neoclassical assumptions was experiencing transitional dynamics towards steady state. Calculate analytically, the rate of growth of output per labor in this economy.

b) Now suppose that in “midway” to the steady state an earthquake occurred and a portion of capital; stock is destroyed. How is the economy’s steady state and transitional dynamic growth affected? Is the new rate of growth faster or slower in comparison to the pre-earth quake situation?

6) (25 points)

Consider the following economy. Assume that per capita output, y , is given by the Cobb-Douglas function, with

$$Y = f(k, g) = k^{\alpha} g^{1-\alpha}$$

where k is the capital labor ratio and g is government spending per labor.

Labor is increasing in supply at the rate of n ; and there is no technological growth.

Investment per labor accumulates given the law of motion:

$$I / L = s(1-t) f(k, g)$$

where $f(k, g)$ is per capita production function, s , and t is the tax rate to finance government expenditures per capita, g . Government budget is in balance, thus, in per capita terms:

$$g = t f(k, g)$$

a) Derive the rule for the rate of capital accumulation for this economy. What is the steady state growth rate of k ? Show the transition dynamics and the steady state in a graph.

b) How does the steady state rate of growth of k depend on t ? Find the growth rate maximizing value of the tax rate, t in this economy.

Theories of Growth and Development I

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1) In the country of Douglasia, output is produced along the production function:

$$Q = k^\alpha L^{\alpha-1}$$

where Q is output, and K and L are the inputs of capital and labor, respectively.

i. Write the output per labor function for Douglasia. Find an expression of the rental rate of capital (the profit rate) and the wage rate, and the factor shares in terms of the per capita production function parameters.

ii. Suppose that the growth rate of labor force is zero, and the rate of depreciation of capital is 2%. The Douglasians are currently saving 20% of their national output for investment purposes. Further suppose that α is a known parameter estimated to be 0.5. Calculate the steady state capital-labor ratio for Douglasia. Sketch a graph.

iii. Calculate the per capita maximizing rate of savings for Douglasia. Find the levels of per capita consumption, savings, and the capital labor ratio under the new steady state.

iv. Calculate the Golden Age of this system. That is find the per capita profit maximizing levels of per capita consumption, savings, and the capital labor ratio under the new steady state, and compare them with the findings in iii, that is the Golden Rule.

v. Now suppose that a group of young graduates of Bilkent Economics has come with a non-refutable finding that in Douglasia the share of capital, i.e. α , is actually 1.00. Calculate the new steady state and draw the “transitional dynamics” of the economy under this new specification.

2) One of the classical propositions of the *Harrod-Domar* model is that, for balanced growth to exist, the following relationship must hold: $v = s/n$; where v is the capital output ratio, s is savings rate, and n is the population growth rate. Show that the neoclassical growth model satisfies this condition by adjustments in the capital labor ratio in its adjustments towards steady state.

3) Consider the following version of the Neoclassical growth model due to Solow (1956): Output is produced along the production function: $Q = K^\alpha L^{1-\alpha}$ where Q is output, and K and L are the inputs of capital and labor, respectively. Suppose that α is equal to 0.5. Suppose that there is a subsistence level of consumption, c^* , below which consumption will not fall, i.e. the agent will consume all his income, if income falls to c^* . Furthermore, saving rate is regarded not a constant but follows the following path:

For very “low” levels of output and capital, $s = s'$ such that $k = \varepsilon$ ($\varepsilon > 0$), so that a minimal amount of capital can be invested to be able to produce c^* . Beyond this level output, saving rate follows the following rule:

$$s = g(k) \text{ with } 0 < g'(k) < 1 \text{ and } g''(k) < 0 \text{ for } y > c^*;$$

Where s is saving rate per person, and y is per capita income. Sketch out the transitional dynamics towards the steady state with the aid of a graph of output per labor and capital output ratio, and discuss the mechanisms of adjustment. Observe that you are not asked a formal analytical depiction of steady state equilibrium.

Theories of Growth and Development

Prof. Dr. Erinc Yeldan

Consider the following simple Kaldorian economy with no government and no trade: Total output (GNP) denoted by Y , is produced by capitalists and workers. The aggregate consumption function of the economy is

$$C = 0.9 * Y$$

Total desired investment, I , is 500 units.

- Calculate equilibrium level of income Y .
- Suppose that capitalists propensity to save, S_c , is 0.25; and that of workers, S_w , is 0.05. Find the share of profits in income and the rate of profit under the Kaldorian equilibrium. Suppose that capital stock is 1000 units.
- What are the total profit income of capitalists and total wage income of workers in this economy?
Using S_c and S_w out of respective incomes of capitalists and workers verify that total investment in this economy; i.e. , that the economy is in Kaldorian equilibrium.

More excitement

d) Now consider the same model under Pasinetti's 1961 remark that "if workers save ($S_w > 0$), then they must be obtaining a share of profits as additional income, as well". Using the same values for S_c and S_w calculate the capitalists' share of profits as a ratio of total income (P_c/Y).

e) What will be the rate of profit and the share of total profits in income?

Does the rate of profit (or the distribution of total profits and wages in income) depend on the workers' propensity to save? Explain.

In particular, which variable determines the distribution of income between wages and profits in the economy?

Even More Excitement

f) Calculate the workers share of total profits, and the value of total wages. Verify that in this "Pasinettian" economy, when workers save they receive an amount of profits (P_w) such that their total saving out of their total income will exactly be equal to the amount that capitalists would have saved out of workers profits if those profits had remained to them.

Hint: verify equation 17 in Pasinetti, 1961.

Ultimate Excitement

g) In particular, observe that the total values of capitalists' and workers' income are equal to those of the Kaldorian economy. Can you explain this?

Theories of Growth and Development

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Consider the following private ownership economy: There is only one homogenous good, wheat, which is produced using labor and seeds of wheat alone. Land is in abundant supply, and is not considered to be a scarce good.

The total wheat harvested is given by the (neoclassical) production function $g k q =$. Currently $s = ky = 0.4$ units of wheat is saved and invested as seeds for next year's production cycle. In this model, seeds invested in the previous year become capital

stock of the current year. Otherwise capital cannot be stored and accumulated. Thus depreciation of capital, d is 1.0. The population growth rate, is 0. The technology parameter, g , is known to be 0.5.

a) Under neoclassical growth, what would be the steady state equilibrium value of the labor-output ratio, ly , in this economy?

ii) What are the profit rate and the wage rate under neoclassical steady state equilibrium?

(iii) Verify the unit cost price equation: $1 = w ly + (1+r) ky$

b) Under neoclassical steady state equilibrium,

(i) What is the value of embodied labor per unit of wheat produced?

(ii) What is the surplus value?

(iii) What is the Marxian rate of exploitation? Is there any exploitation in the neoclassical sense? Discuss analytically.

c) Derive analytically a rule for maximizing per capita consumption in this economy, i.e. The Golden Rule of Accumulation.

(i) Calculate the Marxian rate of exploitation under the Golden Rule of Accumulation. Observe that the saving rate, s , is also equal to the seed requirement rate (capital output ratio) ky . Thus in this part of the question, ky should be changed to the new value of s^{**} . Is the new exploitation rate higher, lower or the same? How would you explain the new value of the exploitation rate?

d) Assuming that all wages are consumed (that is $W = C$), and considering that in this economy capital stock is equal to annual investment ($K = I$) calculate the Neo-Keynesian (Kaldor-Pasinetti) equilibrium values of the profits and the share of wages (per capita).

e) Show the Marxian equilibrium of this economy in a 4-quadrant graph. Sketch a procedure showing how you would derive the Keynesian equilibrium of this economy. Again make additional assumptions if necessary.